

# MAGNETO-OPTIC KERR EFFECT

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The Magneto-Optic Kerr Effect (MOKE) is the study of the reflection of polarized light by a material sample subjected to a magnetic field. This reflection can produce several effects, including 1) rotation of the direction of polarization of the light, 2) introduction of ellipticity in the reflected beam and 3) a change in the intensity of the reflected beam. MOKE is particularly important in the study of ferromagnetic and ferrimagnetic films and materials.

## Geometry of MOKE Experiments

There are three “geometries” for MOKE experiments, the POLAR, LONGITUDINAL and TRANSVERSE geometries. These arise from the direction of the magnetic field with respect to the plane of incidence and the sample surface. These relationships are summarized in Table I.

TABLE I: DEFINITION OF MOKE GEOMETRIES

GEOMETRY	MAGNETIC FIELD ORIENTATION	OBSERVABLE AT NORMAL INCIDENCE?	DIAGRAM
POLAR	Parallel to the plane of incidence, normal to the sample surface	Yes	Figure 1
LONGITUDINAL	Parallel to the plane of incidence and the sample surface	No	Figure 4
TRANSVERSE	Normal to the plane of incidence, parallel to the sample surface	No	Figure 5

In the polar and longitudinal cases, MOKE induces a Kerr rotation  $\theta_k$  and an ellipticity  $\epsilon_k$ .<sup>1</sup>

## Polar MOKE

For polar MOKE, the magnetic vector is parallel to the plane of incidence and normal to the reflecting surface. (Figure 1.)

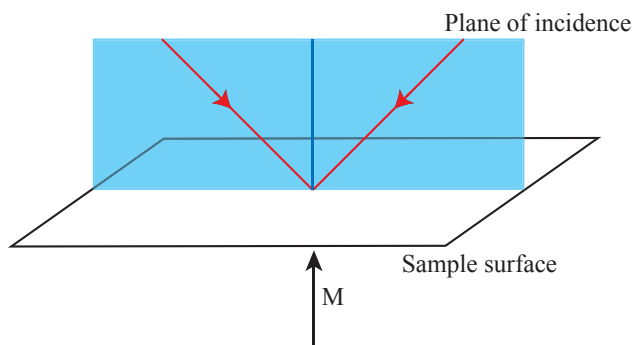


Figure 1. Polar MOKE geometry

Polar MOKE is most frequently studied at near-normal angles of incidence and

reflection to the reflecting surface. A practical reason is that both beams must pass through a hole in one pole of the magnet. The polar geometry is the only one where MOKE can be observed at normal incidence.

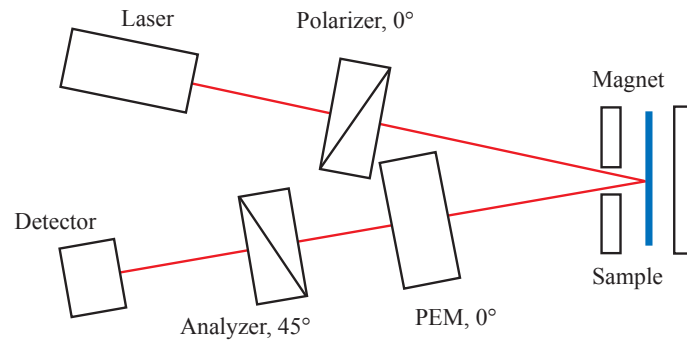


Figure 2. Optical Bench Setup for Polar MOKE

Atkinson<sup>2</sup> has shown one experimental setup for polar MOKE. (Figure 2)

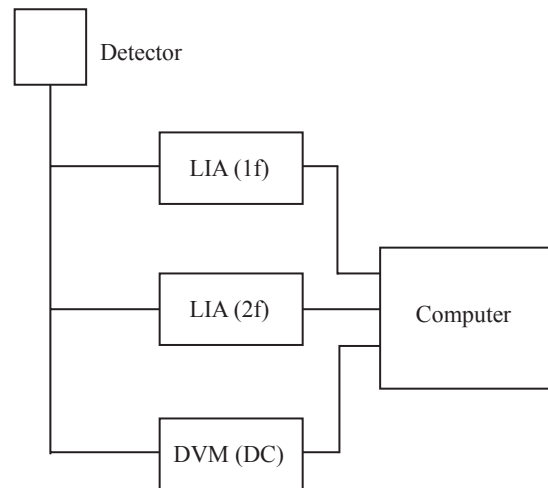


Figure 3. Electronic block diagram for MOKE

After passing through the analyzer to a detector, the detector signal goes to two lock-in amplifiers and a digital voltmeter, then to a computer. (Figure 3) Some experimenters put the 45° polarizer and the PEM at 0° before the sample and the analyzer polarizer at 0° after the sample.<sup>3</sup>

The operation of this system may be analyzed using Mueller matrices and Fourier series. The Kerr rotation angle  $\theta_k$  and the Kerr ellipticity  $\epsilon_k$  are typically very small numbers, therefore small angle approximations may be used. The intensity of light (as a function of time) arriving at the

detector may be calculated. (See Appendix A for the derivation.)

$$I(t) = I_0[1 + 2\theta \cos(A_0\omega t) - 2\epsilon_k \sin(A_0\omega t)] \quad (1)$$

where  $I_0$  represents the "average" or DC intensity,  $\omega = 2\pi f$  is the angular frequency of the PEM oscillations and  $A_0$ , the retardation amplitude of the PEM. Using a Fourier series expansion to represent this equation and keeping only the first three terms,

$$I(t) \cong I_0[1 + 2\theta_k J_0(A_0) - 4\epsilon_k J_1(A_0) \sin(\omega t) + 4\theta_k J_2(A_0) \cos(2\omega t)] \quad (2)$$

The second term is part of the DC term and may be neglected for either or both of two reasons: 1)  $\theta_k$  is a very small number and/or 2)  $A_0 = 2.405$  radians, in which case  $J_0 = 0$ .

Three voltages are measured:  $V_{DC}$ ,  $V_{1f}$  and  $V_{2f}$ . It is traditional to form the ratios of the AC term to the DC term, since by doing so the experiment becomes immune to fluctuations in the light intensity, changes in transmission, etc. Thus

$$\theta_k = \frac{\sqrt{2}}{4J_2} \frac{V_{2f}}{V_{DC}} \quad [3] \quad \text{and} \quad \varepsilon_k = \frac{\sqrt{2}}{4J_1} \frac{V_{1f}}{V_{DC}} \quad [4]$$

The factor  $\sqrt{2}$  arises since lock-in amplifiers display the rms voltage, whereas the theory is written in terms of peak voltage or voltage amplitude.

### Longitudinal MOKE

For longitudinal MOKE, the magnetic field is parallel to both the plane of incidence and the sample surface. (Figure 4)

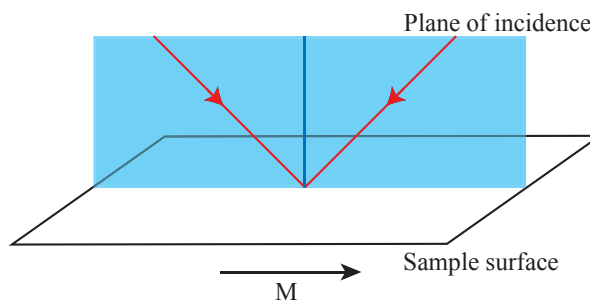


Figure 4. Longitudinal MOKE geometry

The experimental setup is similar to the case of polar MOKE, except that care must be taken with the relative orientations of the PEM and the polarizers with respect to the plane of incidence. The PEM retardation axis is typically normal to the plane of incidence although it can be parallel to the plane of incidence as well. The analyzer polarizer is oriented at  $45^\circ$  with respect to the PEM retardation axis.

There are two cases of incident polarization that must be considered. These are: the S polarization, where the direction of the linear polarization is normal to the plane of incidence, and the P-polarization, where the plane of polarization and the plane of incidence are parallel. If the first polarizer is placed in a precision rotator, both cases may be studied by simply changing the first polarizer orientation.

For the longitudinal MOKE setup, the equations for polar MOKE apply.<sup>1</sup>

### Transverse MOKE

For the transverse MOKE geometry, the magnetic field is normal to the plane of incidence. (Figure 5)

For the transverse MOKE geometry, only the P-polarization shows an effect, and this is quite different from the polar MOKE and longitudinal MOKE effects. A small Kerr vector  $\mathbf{k}$  is generated which is parallel to the reflected polarization. Whether this increases or decreases the polarization amplitude depends on the direction of the magnetic field.<sup>1</sup>

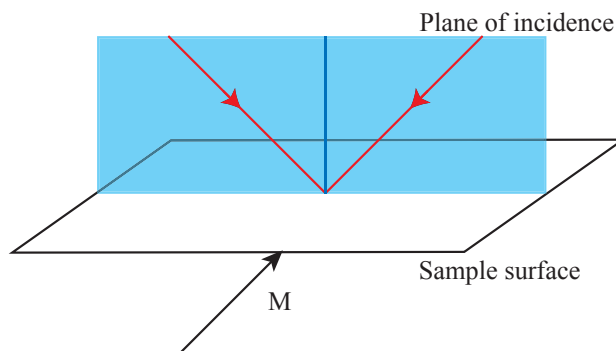


Figure 5. Transverse MOKE geometry

If very fast response is required, a PEM could be used as an optical chopper to modulate the incident laser beam. AC techniques (e.g. a lock-in amplifier) could then be used to measure the intensity of the reflected beam.

### Modulated Interference Effects

Lasers are typically used for MOKE experiments. When lasers are used with PEMs an effect called "modulated interference" may be present.<sup>4</sup> Since the MOKE AC signals are typically very weak, great care must be taken to eliminate or reduce these effects. Useful techniques include: 1) use of anti-reflective (AR) coatings on the PEM optical element, 2) use of a PEM with a slight wedge angle (non-parallel optical surfaces) or 3) tilting the PEM so that the laser beam is not incident on the optical element at a normal angle. More detail is given in Newsletter #8 which can be found on the Hinds Instruments website.

### References

1. [www.qub.ac.uk/mp/con/magnetics-group/magnetooptics.html](http://www.qub.ac.uk/mp/con/magnetics-group/magnetooptics.html)
2. Atkinson, *Magnetism in a New Light*, Hinds Instruments Newsletter No. 10, 2001. [www.hindsinstruments.com/media/pem-10.pdf](http://www.hindsinstruments.com/media/pem-10.pdf)
3. [www.amn.uno.edu/kstokes/mo.html](http://www.amn.uno.edu/kstokes/mo.html)
4. Oakberg, *Modulated Interference Effects in Photoelastic Modulators*, Hinds Instruments Newsletter No. 8, 1998. [www.hindsinstruments.com/media/pem-8.pdf](http://www.hindsinstruments.com/media/pem-8.pdf)
5. Kliger, Lewis and Randall, *Polarized Light in Optics and Spectroscopy*, Academic Press, 1990.

## Appendix A

### Intensity vs. Time For Longitudinal and Polar MOKE

In polar and longitudinal MOKE, the polarization state of the reflected light has been altered. Specifically, an ellipticity  $\epsilon_k$  has been introduced and the plane of polarization (major axis of the ellipse) has been rotated by an angle  $\theta_k$ . Both  $\theta_k$  and  $\epsilon_k$  are typically very small numbers, therefore small angle approximations will be used in this analysis.

The light incident on the sample is linear polarized at 0 degrees with the Stokes vector

$$S_{\text{incident}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (\text{A1})$$

Upon reflection, the plane of polarization has been rotated  $\theta_k$  and ellipticity  $\epsilon_k$  has been introduced. We wish to determine the new Stokes vector  $S_{\text{reflected}}$ .

From Kliger, Lewis and Randall, equations 5.48 and 5.49<sup>5</sup>

$$\epsilon = \tan \left[ \frac{1}{2} \sin^{-1} \left( \frac{V}{I} \right) \right] \approx \frac{1}{2} \frac{V}{I} = \frac{v}{2} \quad (\text{A2})$$

and

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right) \approx \frac{1}{2} \frac{U}{Q} = \frac{1}{2} \frac{u}{q} \quad (\text{A3})$$

Where I, Q, U, V are the components of the Stokes vector and q, u, and v are the normalized components where  $q = Q/I$ , etc. In the incident beam,  $Q = I$  or  $q = 1$ . To first order, this is maintained in the reflected beam, since  $\epsilon_k$  and  $\theta_k$  are very small. We may therefore solve equations A2 and A3 for u and v.

$$u \approx 2\theta_k \quad (\text{A4}) \quad v \approx -2\epsilon_k \quad (\text{A5})$$

The minus sign in (A5) relates the sense of the elliptical polarization and the angular direction of the rotation. The Stokes vector for the reflected light is therefore

$$S_{\text{incident}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (\text{A6})$$

Continuing the Mueller matrix analysis of MOKE

The Mueller matrix for the PEM at 0 degrees is:

$$M(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(A) & \sin(A) \\ 0 & 0 & -\sin(A) & \cos(A) \end{bmatrix} \quad (A7)$$

Where  $A = A_0 \sin(\omega t)$  is the time-dependent retardation of the PEM.

The Stokes vector of the light following the PEM is:

$$Sb(\theta_k, \epsilon_k, A) = M(A) \cdot Sa(\theta_k, \epsilon_k) \rightarrow \begin{bmatrix} 1 \\ 1 \\ 2 \cdot \cos(A) \cdot \theta_k - 2 \cdot \sin(A) \cdot \epsilon_k \\ -2 \cdot \sin(A) \cdot \theta_k - 2 \cdot \cos(A) \cdot \epsilon_k \end{bmatrix} \quad (A8)$$

The next component is a linear polarizer at +45 degrees.

$$Mc = .5 \cdot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (A9)$$

The Stokes vector of the light arriving at the detector is:

$$Sc(\theta_k, \epsilon_k, A) = Mc \cdot Sb(\theta_k, \epsilon_k, A) \rightarrow \begin{bmatrix} .5 + 1.0 \cdot \cos(A) \cdot \theta_k - 1.0 \cdot \sin(A) \cdot \epsilon_k \\ 0 \\ .5 + 1.0 \cdot \cos(A) \cdot \theta_k - 1.0 \cdot \sin(A) \cdot \epsilon_k \\ 0 \end{bmatrix} \quad (A10)$$

The top line of the final matrix is proportional to the intensity at the detector vs. time. Defining  $I_0$  as the "average" or DC intensity, we may write

$$I(t) = I_0 [1 + 2\theta \cos(A_0 \omega t) - 2\epsilon_k \sin(A_0 \omega t)] \quad (A11)$$

Expanding this as a Fourier series and keeping only the first few terms

$$I(t) \cong I_0 [1 + 2\theta_k J_0(A_0) - 4\epsilon_k J_1(A_0) \sin(\omega t) + 4\theta_k J_2 \cos(2\omega t) + \dots] \quad (A12)$$

The second term may be neglected: 1) because  $\theta_k$  is a very small number and/or  $A_0 = 2.405$  radians in which case  $J_0 = 0$ . The latter is a good choice for  $A_0$ , since the sensitivities to  $\theta_k$  and  $\epsilon_k$  will be approximately equal.